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Two-Dimensional Lorentz–Weyl Anomaly and Gravitational Chern–Simons Theory

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Abstract. Two-dimensional chiral fermions and bosons, more generally conformal blocks of two-dimensional conformal field theories, exhibit Weyl-, Lorentz- and mixed Lorentz–Weyl anomalies. A novel way of computing these anomalies for a system of chiral bosons of arbitrary conformal spin j is sketched. It is shown that the Lorentz- and mixed Lorentz–Weyl anomalies of these theories can be cancelled by the anomalies of a three-dimensional classical Chern–Simons action for the spin connection, expressed in terms of the dreibein field. Some tentative applications of this result to string theory are indicated.

There are two circles of problems in theoretical physics which lead us to reconsider some aspects of two-dimensional chiral anomalies. The first one concerns the theory of incompressible, chiral quantum fluids, in particular of two-dimensional electron fluids in a transverse, external magnetic field encountered in studies of the quantized Hall effect. The study of the dynamics of Hall fluids near the boundary of the system naturally leads one to consider $(1 + 1)$-dimensional abelian and non-abelian gauge anomalies and the associated abelian and non-abelian Chern–Simons gauge theories in $2 + 1$ dimensions which describe bulk properties of two-dimensional, incompressible, chiral quantum fluids in the large-scale, low-frequency limit [1, 2].

The second circle of problems concerns the anomalies, in particular the Lorentz- and Weyl anomalies, of two-dimensional (chiral) conformal field theory and the problem of constructing new string theories [3–5].

This note has grown out of studying these two circles of problems. It is quite likely that the following calculations and remarks are known to experts in the field. Nevertheless, we wish to submit the results of a series of exercises that we have performed to the attention of the interested reader.

Chiral fermions coupled to an external gravitational field in $(2 + 4n)$ dimensions, $n = 0, 1, 2, \ldots$, are known to exhibit Weyl- and Lorentz anomalies [6–8]. At the classical level, the action is invariant under Weyl transformations and Lorentz transformations of the local Lorentz frames. However, quantization of the fermions breaks these symmetries. The effective action, defined as the logarithm of the chiral-, or Weyl determinant, is *not* invariant under Weyl- and local Lorentz