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Hidden Symmetry Breaking and the Haldane Phase in $S = 1$ Quantum Spin Chains

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Abstract. We study the phase diagram of $S = 1$ antiferromagnetic chains with particular emphasis on the Haldane phase. The hidden symmetry breaking measured by the string order parameter of den Nijs and Rommelse can be transformed into an explicit breaking of a $Z_2 \times Z_2$ symmetry by a nonlocal unitary transformation of the chain. For a particular class of Hamiltonians which includes the usual Heisenberg Hamiltonian, we prove that the usual Néel order parameter is always less than or equal to the string order parameter. We give a general treatment of rigorous perturbation theory for the ground state of quantum spin systems which are small perturbations of diagonal Hamiltonians. We then extend this rigorous perturbation theory to a class of "diagonally dominant" Hamiltonians. Using this theory we prove the existence of the Haldane phase in an open subset of the parameter space of a particular class of Hamiltonians by showing that the string order parameter does not vanish and the hidden $Z_2 \times Z_2$ symmetry is completely broken. While this open subset does not include the usual Heisenberg Hamiltonian, it does include models other than VBS models.

1. Introduction

Much of our intuition for quantum spin systems is based on our understanding of the corresponding classical systems. However, occasionally the quantum spin systems surprise us. One of the most interesting surprises is the qualitative dependence of the properties of the one dimensional Heisenberg antiferromagnet on whether the spin is integral or half integral which was discovered by Haldane. He argued that the ground state has an excitation gap and exponentially decaying correlation functions when S is integral, while it has a ground state without a gap and correlation functions with power law decay when S is half integral [22]. Haldane's argument was based on an approximate mapping of the spin chain onto a two dimensional quantum field theory.

The Heisenberg antiferromagnet has a continuous symmetry; we can rotate all the spins by the same amount, and the Hamiltonian will be unchanged. This suggests that it should be possible to construct excitations with arbitrarily low