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The Pattern of Escape from Metastability of a Stochastic Ising Model*

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Abstract. We study further the metastable behavior of Metropolis dynamics for the two-dimensional nearest neighbor ferromagnetic Ising model, with positive and small external field, in the limit as the temperature vanishes (see [NS]). We focus on the typical features of the escape (nucleation) from the (metastable) configuration with all spins -1, to the (stable) configuration with all spins $+1$. Using the reversibility of the process as the main tool, we prove (for the discrete time version of the model) that the first step of a typical escaping path is the time reverse of a typical time evolution of a shrinking subcritical rectangular droplet, which is one slice smaller than a critical droplet. This subcritical droplet then evolves in a time of order 1 to a critical droplet, which finally grows with features described in [NS].

I. The Model

In this paper we consider a discrete time version of Metropolis dynamics for the two-dimensional nearest neighbor ferromagnetic Ising model. The state space of the process is the set $\{-1, +1\}^{\Lambda_N}$, where $\Lambda_N = \{1, \ldots, N\}^2$ is taken with periodic boundary conditions (meaning that two sites $x, y \in \Lambda_N$ will be said to be neighbors if they have one coordinate in common and the other one differing by 1 or $N-1$). $\sigma(x) = \pm 1$ is called the spin at the site x. To each configuration $\sigma \in \{-1, +1\}^{\Lambda_N}$ an energy is assigned by the expression

$$H(\sigma) = -\frac{1}{2} \sum_{x, y \in \mathbb{B}_N} \sigma(x)\sigma(y) - \frac{h}{2} \sum_{z \in \Lambda_N} \sigma(z) ,$$

where $\mathbb{B}_N$ is the set of (unordered) pairs of neighbors in Λ_N and h is called the external field. The discrete time Metropolis dynamics at inverse temperature $\beta > 0$ is the Markov chain defined by taking uniformly at random at each integer unit of time a site $x \in \Lambda_N$ and then flipping the spin there with probability

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