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The Faddeev–Popov Procedure and Application to Bosonic Strings:

An Infinite Dimensional Point of View

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Abstract. A generalisation of the finite dimensional presentation of the Faddeev–Popov procedure is derived in an infinite dimensional framework for gauge theories with finite dimensional moduli space using heat-kernel regularised determinants. It is shown that the infinite dimensional Faddeev–Popov determinant is – up to a finite dimensional determinant determined by a choice of a slice – canonically determined by the geometrical data defining the gauge theory, namely a fibre bundle $P \to P/G$ with structure group G and the invariance group of a metric structure given on the total space P. The case of (closed) bosonic string theory is discussed.

0. Introduction

The Faddeev–Popov procedure for gauge theories originally introduced by Faddeev and Popov in the context of Yang-Mills theories [1] has been discussed by many authors in the physics literature in the context of string theory (see e.g. [2]) from a topological point of view (see e.g. [3–5]) as well as from a geometrical stand-point (see e.g. [6–8]). It essentially yields a formal procedure to write a functional integral on the space P of paths arising from the functional quantisation of a classical action invariant under the action of the gauge group G as an integral on the quotient space P/G (or a submanifold Σ of P isomorphic to this quotient). If the quotient space is finite dimensional as in the case of bosonic string theory (it is given by the Teichmüller space of a Riemann surface), this procedure reduces a formal integration on an infinite dimensional space, the space of configurations to an integration on a finite dimensional manifold. "Factorising out" the gauge group in this way gives rise to a jacobian determinant, the formal Faddeev–Popov determinant. Some important clarifications were made as to the geometrical meaning behind this formal procedure [6, 7]. This geometrical interpretation was done in a finite dimensional setting with the implicit point of view that the infinite dimensional set up inherent to functional integration can be seen as a generalisation.

In this paper, we want to discuss how far this generalisation to an infinite dimensional framework can be made precise from a mathematical point of view.