Commun. Math. Phys. 146, 483–504 (1992)

Communications in
Mathematical
Physics

An Extension of the Borel–Weil Construction to the Quantum Group $U_q(n)$*

L. C. Biedenharn[1] and M. A. Lohe[2]

[1] Department of Physics, Duke University, Durham, NC 27706, USA
[2] Faculty of Science, Northern Territory University, Casuarina, NT 0811, Australia

Received July 4, 1991

Abstract. The Borel–Weil (BW) construction for unitary irreps of a compact Lie group is extended to a construction of all unitary irreps of the quantum group $U_q(n)$. This q-BW construction uses a recursion procedure for $U_q(n)$ in which the fiber of the bundle carries an irrep of $U_q(n-1) \times U_q(1)$ with sections that are holomorphic functions in the homogeneous space $U_q(n)/U_q(n-1) \times U_q(1)$. Explicit results are obtained for the $U_q(n)$ irreps and for the related isomorphism of quantum group algebras.

1. Introduction

There is an elegant geometric procedure for constructing all unitary irreducible representations (irreps) of a compact classical Lie group, the Borel–Weil (BW) construction [8] which realizes irreps as holomorphic sections of homogeneous holomorphic line bundles over Kähler manifolds. Our objective in the present paper is to develop the quantum group extension of this construction for the quantum group $U_q(n)$, a q-deformation of the enveloping algebra of the classical Lie algebra $A_{n-1} \times A_0$.

The Borel–Weil method [14, 24], applied to representations of a compact simple Lie group G, constructs a line bundle over the homogeneous space G/T, where T is the maximal torus of G. This coset space G/T can be made into a complex manifold, as can be seen from the fact that $G/T \cong G_c/B^+$, where G_c is the complexified group G and B^+ is the Borel sub-group. (We will be considering primarily $U(n)$ for which B^+ is the sub-group of upper triangular matrices.) To every character λ of the torus T one associates a homogeneous holomorphic line bundle L_λ over G/T; this uses the fact that every homomorphism λ: $T \to \mathbb{C}^x$, extends uniquely to a holomorphic homomorphism $\lambda: B^+ \to \mathbb{C}^x$, so that one may define the associated line bundle

* Supported in part by the National Science Foundation, No. PHY-9008007