

Non-Self-Dual Yang-Mills Connections with Quadrupole Symmetry

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Abstract. We prove the existence of non-self-dual Yang-Mills connections on $SU(2)$ bundles over the four-sphere, specifically on all bundles with second Chern number not equal ± 1 . We study connections equivariant under an $SU(2)$ symmetry group to reduce the effective dimensionality from four to one, and then use variational techniques. The existence of non-self-dual $SU(2)$ YM connections on the trivial bundle (second Chern number equals zero) has already been established by Sibner, Sibner, and Uhlenbeck via different methods.

1. Introduction

1.1. Background and Statement of the Main Result

In this paper we prove the existence of non-self-dual Yang-Mills connections on topologically nontrivial $SU(2)$ bundles over the four-sphere S^4 , with the standard metric. For brevity, we use the term *non-self-dual* to refer to connections that are neither self-dual nor anti-self-dual. Recall that the $SU(2)$ bundles over S^4 are topologically classified by C_2 , the second Chern number of the associated vector bundles. The existence of non-self-dual Yang-Mills connections on the trivial $SU(2)$ bundle over S^4 has been proven by Sibner, Sibner, and Uhlenbeck [SSU]. As announced in [SS2], we prove that non-self-dual YM connections exist on all $SU(2)$ bundles with $C_2 \neq \pm 1$. Existence for $C_2 = \pm 1$ is still an open problem.

The study of non-self-dual Yang-Mills connections has developed rather slowly compared to the progress made in the understanding of self-dual connections. The first nontrivial solution of the Yang-Mills equations on S^4 was the self-dual $SU(2)$ instanton, found in 1975 [BPST]. Three years later, the self-dual solutions on S^4 were

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