Commun. Math. Phys. 145, 345–356 (1992)

Communications in Mathematical Physics

Solutions Without Phase-Slip for the Ginsburg–Landau Equation

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Received October 28, 1991

Abstract. We consider the Ginsburg–Landau equation for a complex scalar field in one dimension and consider initial data which have two different stationary solutions as their limits in space as $x \to \pm\infty$. If these solutions are not very different, then we show that the initial data will evolve to a stationary solution by a "phase melting" process which avoids "phase slips," i.e., which does not go through zero amplitude.

1. Introduction

In this paper, we pursue our study of the Ginsburg–Landau equation

$$\partial_t u = \partial_x^2 u + u - u|u|^2, \tag{1.1}$$

where $u\colon \mathbf{R} \times \mathbf{R} \to \mathbf{C}$, cf. [CE, CEE]. We shall fill in more details of the phase diagram of this equation, by studying the time evolution for initial data which are close to stationary with *different* amplitudes at $\pm\infty$. More precisely, define two stationary solutions $u_\pm$ by

$$u_\pm(x) = r_\pm e^{iq_\pm x + i\theta_\pm}, \tag{1.2}$$

with $r_+ = (1 - q_+^2)^{1/2}, r_- = (1 - q_-^2)^{1/2}$. Assume now that the initial data u_0 satisfy

$$\lim_{x \to \pm\infty} u_0(x) - u_\pm(x) = 0,$$

in a sense to be described in more detail below, and assume $r_\pm \approx 1$.

Under these conditions, see below for details, we shall show that the solutions have *no phase slips*. See Langer and Ambegaokar [LA] for an example with phase slips. Furthermore, we will show convergence to a "stationary" solution in the sense that $u(x,t) = r(x,t)e^{i\phi(x,t)}$ satisfies

$$\sup_{x\in\mathbf{R}} |r'(x,t)| \leqq \varepsilon, \qquad \sup_{x\in\mathbf{R}} |\phi''(x,t)| \leqq \varepsilon,$$