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Space-Time Picture of Semiclassical Resonances

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Abstract. This paper addresses the theory of quasiclassical resonances for Schrödinger operators with potentials smooth outside some possible local singularities. We introduce the notion of quasiresonance similar to that of quasimode, but incorporating a condition revealing its scattering nature, and describe its space–time behaviour. The definition is given in terms of the original Schrödinger operator and uses a description of its frequency set. The result on the space-time behaviour justifies the intuitive picture of resonances as metastable states or "bound states with finite life-times." We demonstrate how quasiresonances arise in several natural situations.

1. Introduction

The quantum resonance is one of the central notions in Modern Physics. However, its mathematical understanding is still at a preliminary stage. The formal definition of the resonance in terms of the poles of a meromorphic continuation of the S-matrix or in terms of bumps in the scattering cross section, given in Physics, is hard to study. In Mathematical Physics the resonances are defined as complex eigenvalues of quantum Hamiltonians deformed by complex canonical transformations. This approach was developed in successive degrees of generality in [Ag–Co, Ba–Co, Si2, S1, Hu1, Cy, He–Sj2]. One knows that both definitions yield the same object for two-body potentials (see for example [Ba1, Ge–Ma1]) and in some cases for N-body potentials (see [Ba2, Ha, De, S3]). Both definitions require analytic (either in the coordinate or momentum representation) potentials, use more complicated objects than original Hamiltonians and are not accessible for phase–space analysis ([He–Sj2] uses a phase–space analysis but not in the original phase–space).

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