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# Hamiltonian Formalism of Whitham-Type Hierarchies and Topological Landau-Ginsburg Models

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**Abstract.** We show that the bi-hamiltonian structure of the averaged Gelfand-Dikii hierarchy is involved in the Landau-Ginsburg topological models (for $A_n$-Series): the Casimirs for the first P.B. give the correct coupling parameters for the perturbed topological minimal model; the correspondence {coupling parameters} $\to$ {primary fields} is determined by the second P.B. The partition function (at the tree level) and the chiral algebra for LG models are calculated for any genus $g$.

## Introduction

We start with explanation of the term "Whitham-type hierarchy" and with brief summary of the Landau-Ginsburg potential formalism in topological minimal models.

*Whitham-type hierarchy.* Let

$$\partial_{t_a}\psi = F_a(\psi, \psi_x, \dots), \qquad a = 1, 2, \dots \tag{0.1}$$

be a KdV-type hierarchy of pairwise commuting evolutionary systems. Let us fix a $N$-dimensional family of invariant $m$-tori. In other words we fix a family of exact solutions of (0.1) of the form

$$\psi = \Psi(t_1\kappa^{(1)} + t_2\kappa^{(2)} + \cdots + \varphi^0; u^1, \dots, u^N) \tag{0.2}$$

(let $F_1 = \psi_x$ so $t_1 \equiv x$). Here $\Psi = \Psi(\varphi_1, \dots, \varphi_m; u^1, \dots, u^N)$ is a $2\pi$-periodic in $\varphi_1, \dots, \varphi_m$ function depending on the parameters $u = (u^1, \dots, u^N)$; $\kappa^{(a)} = (\kappa_1^{(a)}(u), \dots, \kappa_m^{(a)}(u))$; $\varphi^0 = (\varphi_1^0, \dots, \varphi_m^0)$ is an arbitrary phase shift. The parameters $u = (u^1, \dots, u^N)$ belong to a $N$-dimensional manifold $M$. In the nonlinear WKB-approximation [1] ("Whitham averaging method") the hierarchy (0.2)

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