

# Generalized Drinfel'd-Sokolov Hierarchies

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**Abstract.** A general approach is adopted to the construction of integrable hierarchies of partial differential equations. A series of hierarchies associated to untwisted Kac–Moody algebras, and conjugacy classes of the Weyl group of the underlying finite Lie algebra, is obtained. The generalized KdV hierarchies of V.G. Drinfel'd and V.V. Sokolov are obtained as the special case for the Coxeter element. Various examples of the general formalism are treated in some detail; including the fractional KdV hierarchies.

## 1. Introduction

It is only comparatively recently that some degree of mastery of non-linear equations has been achieved, and in this regard it is the notion of integrability that has proved one of the most useful concepts. Prior to the discovery of the Korteweg de Vries (KdV) and Nonlinear Schrödinger (NLS) equations, there were very few known integrable systems, examples being the harmonic oscillator and rigid body motion. (There was also the exactly solvable Ising model of Statistical Mechanics, the only such system with an infinite number of degrees of freedom.) In the 1960's the KdV equation was shown to have an infinite number of conservation laws and in fact to be integrable (see [1] for a nice discussion of the history of these and subsequent developments). The existence of soliton solutions of the KdV equation, which is a hallmark of integrable systems, is the result of an apparently delicate interplay between dispersion on the one hand and nonlinearity on the other – the latter causing a steepening of the wave-front. Despite the seemingly delicate nature of the interplay required, the KdV equation finds a wide application in physics. Five years after the efforts required to demonstrate the integrability of this equation, Zakharov and Shabat found the Lax pair formulation of the NLS equation, and shortly thereafter the sine-Gordon equation was similarly treated (these equations too are widespread). Subsequent to the successful treatment of the above three equation, all sorts of generalizations were obtained [1].