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Classical $N = 1$ W-Superalgebras from Hamiltonian Reduction

José M. Figueroa-O'Farrill* and Eduardo Ramos**

Instituut voor Theoretische Fysica, Universiteit Leuven, Celestijnenlaan 200D, B-3001 Heverlee, Belgium

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Abstract. A combinatorial proof is presented of the fact that the space of supersymmetric Lax operators admits a Poisson structure analogous to the second Gel'fand–Dickey bracket of the generalized KdV hierarchies. This allows us to prove that the space of Lax operators of odd order has a symplectic submanifold – defined by (anty)symmetric operators – which inherits a Poisson structure defining classical W-superalgebras extending the $N = 1$ supervirasoro algebra. This construction thus yields an infinite series of extended superconformal algebras.

1. Introduction

The study of W-algebras is becoming increasingly relevant in two-dimensional conformal field theory, string theory, and quantum gravity and a lot of the progress in the study of both their classical and quantum versions arises from its connections with the theory of integrable models.

Although quantum W-algebras first make their appearance in the important paper [1] of A. B. Zamolodchikov on extensions of the conformal symmetry of two-dimensional statistical mechanics models, classical W-algebras had already appeared as somewhat exotic hamiltonian structures [2] for the generalized KdV hierarchies. In fact, it was Magri [3] who discovered that the KdV hierarchy was bi-hamiltonian: the second bracket defining a classical version of the Virasoro algebra. The analogous statement of the n^{th} order KdV hierarchy (KdV being $n = 2$) involves the so-called W_n algebra as the "second hamiltonian structure." Efforts to understand the second hamiltonian structure culminated with the discovery by Kupershmidt and Wilson [4, 5] (based on earlier work for the KdV equation by Adler and Moser [6]) of the fact that the second hamiltonian structure was induced

* e-mail: fgbda11@blekul11.BITNET, fgbda11@cc1.kuleuven.ac.be. Address after October 1991: Physikalisches Institut der Universität Bonn, FRG

** e-mail: fgbda06@blekul11.BITNET, fgbda06@cc1.kuleuven.ac.be