Commun. Math. Phys. 145, 1–16 (1992)

Communications in
Mathematical
Physics

Quantization of $SL(2, \mathrm{R})$ Chern–Simons Theory

T. P. Killingback

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Received August 10, 1990; in revised form June 5, 1991

Abstract. We discuss Chern–Simons gauge theory with an $SL(2, \mathbf{R})$ gauge group on an arbitrary 3-manifold M. The $SL(2, \mathbf{R})$ Chern–Simons action is defined for gauge bundles over M of arbitrary topological type. The geometric quantization of $SL(2, \mathbf{R})$ Chern–Simons theory is discussed and related to the quantization of Teichmüller space. The generalization to Chern–Simons theory with an $SL(n, \mathbf{R})$ gauge group is also considered.

1. Introduction

One of the most interesting recent developments in theoretical physics is the realization that there are non-trivial quantum field theories defined on smooth manifolds which are independent of any choice of metric on the manifold. The observables of such a topological quantum field theory are then automatically topological invariants of the situation. The Donaldson invariants of smooth 4-manifolds and the Floer homology groups of 3-manifolds are related to a certain topological gauge theory in $3+1$ dimensions [1]. Similarly the Gromov invariants of symplectic manifolds may be interpreted in terms of the quantum field theory of a topological sigma model in $1+1$ dimensions [2]. In $2+1$ dimensions an interesting topological quantum field theory is defined by the Chern–Simons action [3]. If M is a compact oriented 3-manifold and G is a compact simple simply-connected Lie group, then the partition function of Chern–Simons theory defines a topological invariant of M. If the topology of M is such that the flat connections on M are isolated, then in the semi-classical limit this invariant is related to the Ray–Singer torsion of M. If the manifold M contains an embedded link then the expectation value of the corresponding Wilson lines in M yields an invariant of the link, which in the simplest case is just the Jones polynomial of the link [3].

An important observation made in [3] is that there is a direct connection between Chern–Simons theory in $2+1$ dimensions and conformal field theory in $1+1$ dimensions. The canonical quantization of Chern–Simons theory associates a Hilbert space to a 2-dimensional surface Σ. This Hilbert space may be interpreted