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Toward a Canonical Formalism of Non-Perturbative Two-Dimensional Gravity

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Abstract. On the basis of the previously proposed action principle describing the theory space of 2D gravity in less than one-dimension, we develop a systematic canonical formalism for studying the properties of the string equation in the phase space of the cosmological constant and its canonical conjugate, the puncture operator. The string equation is written in a manifestly invariant form under the group of regular canonical transformations in the phase space of generalized coordinate and momentum. As a consequence, the geometrical origin of the generalized Virasoro condition on the partition function (or more precisely, the τ-function) is understood to be the symmetry under the regular area-preserving diffeomorphisms ($w_{1+\infty}$ symmetry) in the deformed phase space. The deformed canonical formalism can be regarded as a quantization of a classical canonical formalism describing the sphere limit of the theory.

1. Introduction

The recent development [1, 2] of a non-perturbative theory of random surfaces on the basis of the matrix models has provided us a new possibility in studying systems with strongly fluctuating geometries and topologies, without relying upon the intrinsically perturbative world-sheet picture. At present, however, it is not clear whether the matrix models are merely toy models with accidental solvability, or may turn out to exhibit properties of more universal nature which survive in (or are generalized to) realistic quantum gravity and string theories.

It seems therefore important to identify the possible universal framework in which the results of the double scaling limit can be naturally embedded and interpreted. In previous works [3, 4], we have proposed a new action principle which describes the structure of the theory space of 2D gravity, at least, in less-than-one dimension, and studied its symmetry properties. In particular, the Virasoro condition [5] on the τ-function has been understood as a consequence of a conformal symmetry of the action principle.