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A Generalized Spectral Duality Theorem

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Dedicated to Professor Marek Burnat

Abstract. We establish a version of the spectral duality theorem relating the point spectrum of a family of *-representations of a certain covariance algebra to the continuous spectrum of an associated family of *-representations. Using that version, we prove that almost all the images of any element of a certain space of fixed points of some *-automorphism of an irrational rotation algebra via standard *-representations of the algebra in $l^2(\mathbb{Z})$ do not have pure point spectrum over any non-empty open subset of the common spectrum of those images. As another application of the spectral duality theorem, we prove that if almost all the Bloch operators associated with a real almost periodic function on $\mathbb{R}$ have pure point spectrum over a Borel subset of $\mathbb{R}$, then almost all the Schrödinger operators with potentials belonging to the compact hull of the translates of this function have, over the same set, purely continuous spectrum.

Introduction

Let $\Gamma=(\Omega, G, \theta, \mathbb{P})$ be a quadruple consisting of a metrizable compact space Ω; a separable locally compact Abelian group G; a continuous G-action θ on Ω, that is, a mapping $\theta: \Omega \times G \to \Omega$ such that $\theta(\omega, 0)=\omega$ and $\theta(\omega, g+h)=\theta(\theta(\omega, h), g)$ for $\omega \in \Omega$ and $g, h \in G$; and a Borel probability measure $\mathbb{P}$ on Ω that is θ_g-invariant for each $g \in G$, where θ_g is the homeomorphism of Ω given by

$$\theta_g(\omega)=\theta(\omega, g) \qquad (\omega \in \Omega).$$

Hereafter any such Γ will be called a dynamical system. If $\Gamma=(\Omega, G, \theta^{(\alpha)}, m_\Omega)$ is such that Ω is a metrizable compact Abelian group, G is a separable locally compact non-compact Abelian group, $\theta^{(\alpha)}$ has the form

$$\theta^{(\alpha)}(\omega, g)=\omega+\alpha(g) \qquad (\omega \in \Omega, g \in G),$$

where α is a continuous one-to-one homomorphism from G onto a dense subgroup of Ω, and m_Ω is the probabilistic Haar measure on Ω, then Γ will be called a special dynamical system.