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The Dispersionless Lax Equations and Topological Minimal Models

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Abstract. It is shown that perturbed rings of the primary chiral fields of the topological minimal models coincide with some particular solutions of the dispersionless Lax equations. The exact formulae for the tree level partition functions of A_n topological minimal models are found. The Virasoro constraints for the analogue of the τ-function of the dispersionless Lax equation corresponding to these models are proved.

1. Introduction

Topological minimal models were introduced in [1], following general considerations in [2]. They are a twisted version of the discrete series of $N=2$ superconformal Landau-Ginzburg (LG) models. A large class of $N=2$ superconformal LG models have been studied in [3]. It was shown that a finite number of states are topological which means that their operator products have no singularities. These states form a closed ring. For the LG models these rings of the primary chiral fields take the particular simple form and can be represented in terms of the corresponding superpotentials. In the topological twisted version of the superconformal LG models the chiral primary fields are the only physical excitations.

In [4] some evidence was provided that the topological minimal models coupled with topological $2-d$ gravity are equivalent to multi-matrix models. (After [2, 6] it was recognized in [5] that the multi-matrix model corresponds to topological gravity coupled to some matter system, although precisely which system was unknown.) On the other hand the multi-matrix model is believed to be equivalent to ordinary matter coupled to $2-d$ gravity. The last equivalence has led to remarkable connections between the non-perturbative theory of $2-d$ gravity and the theory of the integrable soliton equations [7–9].

Our work was stimulated by the results of [10], where the perturbed rings of the primary chiral fields were found. All correlation functions can be easily obtained