

# Infinite Dimensional Grassmannian Structure of Two-Dimensional Quantum Gravity

Masafumi Fukuma<sup>1, \*</sup>, Hikaru Kawai<sup>1, \*\*</sup>, and Ryuichi Nakayama<sup>2, \*\*\*</sup>

<sup>1</sup> Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

<sup>2</sup> National Laboratory for High Energy Physics (KEK), Tsukuba-shi, Ibaraki 305, Japan

Received March 22, 1991

**Abstract.** We study the infinite dimensional Grassmannian structure of 2D quantum gravity coupled to minimal conformal matters, and show that there exists a large symmetry, the  $W_{1+\infty}$  symmetry. Using this symmetry structure, we prove that the square root of the partition function, which is a  $\tau$  function of the  $p$ -reduced KP hierarchy, satisfies the vacuum condition of the  $W_{1+\infty}$  algebra. We further show that this condition is reduced to the vacuum condition of the  $W_p$  algebra when the redundant variables for the  $p$ -reduction are eliminated. This mechanism also gives a prescription for extracting the  $W_p$  algebra from the  $W_{1+\infty}$  algebra.

## 1. Introduction

Recently great progress has been made in the understanding of the non-perturbative aspects of 2D quantum gravity by formulating it in terms of the matrix models [1]. In particular, it has been recognized that the 2D quantum gravity has a close connection with the KP hierarchy [2–4]. Furthermore, the analysis through the Schwinger-Dyson equation reveals the universal structures of 2D quantum gravity and from this one can obtain an analogue of the operator product expansion even in the theory of quantum gravity [5, 6]. In particular, it is shown [5, 6] that the Schwinger-Dyson equation for the 2D gravity coupled to the  $(p, q)$  conformal matters ( $(p, q)$  quantum gravity)<sup>1</sup> is expressed as the vacuum condition of the  $W_p$  algebra on the function  $\tau(x) = \sqrt{Z(x)}$ , where  $Z(x)$  is the partition function of the 2D gravity with the action  $S = \sum_{n \neq 0 \pmod{p}} x_n \phi_n$  and the  $x$ 's

\* E-mail address: tkyvax\$hepnet:fukuma

\*\* E-mail address: tkyvax\$hepnet:kawai

\*\*\* E-mail address: nakayama@jpnkekvm.bitnet

<sup>1</sup> We use the convention different from that in [4] where the roles of  $p$  and  $q$  are interchanged