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W-Algebras for Generalized Toda Theories

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Abstract. The generalized Toda theories obtained in a previous paper by the conformal reduction of WZNW theories possess a new class of W-algebras, namely the algebras of gauge-invariant polynomials of the reduced theories. An algorithm for the construction of base-elements for the W-algebras of all such generalized Toda theories is found, and the W-algebras for the maximal $SL(N, R)$ generalized Toda theories are constructed explicitly, the primary field basis being identified.

1. Introduction

In some previous papers [1] it was shown that Toda field theories [2] could be regarded as Wess–Zumino–Novikov–Witten (WZNW) theories [3], in which the Kac–Moody (KM) currents were subjected to some first-class linear constraints. Among the advantages obtained by regarding the Toda theories as reduced WZNW theories was a very natural interpretation of the W-algebras [4, 5] of Toda theories, namely, as the algebras of the gauge invariant polynomials of the constrained KM currents and their derivatives [1].

In a subsequent paper [6] it was shown that the WZNW-Toda reduction could be extended to yield a series of generalized Toda theories. These generalized Toda theories are a set of conformally-invariant integrable theories that interpolate between the WZNW theories and the Toda theories, and are partially-ordered in correspondence with the strata of group-orbits in the adjoint representation of the WZNW group G, the traditional Toda theories corresponding to the (unique) minimal stratum. To obtain these generalized Toda theories the KM currents of the WZNW theories are subjected to a more general set of first-class linear constraints, and thus, like the Toda theories, are gauge theories, the gauge group being just that generated by the constraints. As a result these Toda theories possess algebras of gauge-invariant polynomials of the constrained currents and their derivatives, where the multiplication is defined by the Poisson-brackets and commutators of the polynomials in the classical and quantum cases respectively.