

The CPT-Theorem in Two-dimensional Theories of Local Observables

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Abstract. Let \mathcal{M} be a von Neumann algebra with cyclic and separating vector Ω , and let $U(a)$ be a continuous unitary representation of \mathbf{R} with positive generator and Ω as fixed point. If these unitaries induce for positive arguments endomorphisms of \mathcal{M} then the modular group act as dilatations on the group of unitaries. Using this it will be shown that every theory of local observables in two dimensions, which is covariant under translations only, can be imbedded into a theory of local observables covariant under the whole Poincaré group. This theory is also covariant under the CPT-transformation.

I. Introduction

The CPT-theorem played an important role in relativistic quantum field theory. In the beginning of the fifties Schwinger [22] observed that there is an extra symmetry in Lagrange field theory which is now called the CPT-symmetry, i.e. a combination of space-time reflection and charge conjugation. However, this symmetry was not a symmetry in the usual sense, because Schwinger had to reverse the order of factors in the products of field operators. Three years later G. Lüders [16, 17] showed that the CPT-operation is, indeed, an exact symmetry of Lagrange field theory following from the Lorentz-symmetry and the standard assumptions of commutativity and the requirement that the Lagrange-function must formally be selfadjoint. For a detailed discussion see W. Pauli [19]. In 1957 R. Jost [14] gave a proof of the CPT-theorem in the frame of Wightman field theory [26] in which he revealed the connection of the CPT-symmetry with the assumptions of positivity of the energy, Lorentz-invariance, and the standard locality assumptions. In this proof the existence of a vacuum state was essential. But up to now there is no proof of the CPT-theorem in the theory of local observables in the sense of Araki, Haag and Kastler.

Initiated by the result of R. Jost also many results about the connection of spin with statistics have been obtained in the Wightman frame, large parts of