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Summations over Equilaterally Triangulated Surfaces and the Critical String Measure*

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Abstract. We propose a new approach to the summation over dynamically triangulated Riemann surfaces which does not rely on properties of the potential in a matrix model. Instead, we formulate a purely algebraic discretization of critical string path integral. This is combined with a technique which assigns to each equilateral triangulation of a two-dimensional surface a Riemann surface defined over a certain finite extension of the field of rational numbers, i.e. an arithmetic surface. Thus we establish a new formulation in which the sum over randomly triangulated surfaces defines an invariant measure on the moduli space of arithmetic surfaces. It is shown that because of this it is far from obvious that this measure for large genera approximates the measure defined by the continuum theory, i.e. Liouville theory or critical string theory. In low genus this subtlety does not exist. In the case of critical string theory we explicitly compute the volume of the moduli space of arithmetic surfaces in terms of the modular height function and show that for low genus it approximates correctly the continuum measure. We also discuss a continuum limit which bears some resemblance with a double scaling limit in matrix models.

1. Introduction and Summary

Recently it has become clear that sums over certain discretizations of two dimensional surfaces, referred to as dynamical triangulations (DT) may provide a suitable framework for studying (non-perturbative) 2D gravity and eventually

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