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Classical BRST Cohomology and Invariant Functions on Constraint Manifolds. I

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Abstract. Given a manifold M and a submanifold C, together with a Lie group G acting on M and leaving C invariant, it is shown how the algebra of G-invariant functions on C can be described in terms of cohomology when C is defined as the common zero level of an irreducible set of (G-covariant) constraints. The construction is independent of any additional structures such as, e.g., a symplectic structure on M, and therefore it provides a natural framework for a unified description of BRST cohomology both for Lagrangian and Hamiltonian systems. Finally, it is discussed how one can, in various typical situations, replace invariance under an infinite-dimensional gauge group by invariance under a suitable finite-dimensional Lie group; this is a necessary prerequisite for handling BRST cohomology for such systems within a completely finite-dimensional setting.

1. Introduction

Among the various approaches towards the problem of quantizing classical dynamical systems, both in mechanics and in field theory, the method now commonly called BRST quantization is the most recent one. In contrast to other, more traditional techniques such as, e.g., canonical quantization or geometric quantization, the BRST approach is specifically designed to deal with singular dynamical systems, or to be more precise, with dynamical systems whose singular nature is due to the presence of a local (gauge) symmetry (these are the only ones that are of relevance to physics and will in the sequel be simply referred to as gauge theories). The basic idea is to introduce additional unphysical degrees of freedom (ghosts) and to replace the original local (gauge) symmetry by a global (super)symmetry (the BRST symmetry), generated by a single operator (the BRST operator) whose square vanishes and which therefore defines a cohomology theory

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