

# The Quantum Poincaré–Birkhoff–Witt Theorem

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**Abstract.** We define and study a wide class of associative algebras in which the Poincaré–Birkhoff–Witt theorem is valid. This class includes numerous quantum algebras which recently appeared.

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