

Bicovariant Differential Calculus on Quantum Groups $SU_q(N)$ and $SO_q(N)$

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Abstract. Following Woronowicz's proposal the bicovariant differential calculus on the quantum groups $SU_q(N)$ and $SO_q(N)$ is constructed. A systematic construction of bicovariant bimodules by using the \hat{R}_q matrix is presented. The relation between the Hopf algebras generated by the linear functionals relating the left and right multiplication of these bicovariant bimodules, and the q -deformed universal enveloping algebras is given. Imposing the conditions of bicovariance and consistency with the quantum group structure the differential algebras and exterior derivatives are defined. As an application the Maurer-Cartan equations and the q -analogue of the structure constants are formulated.

1. Introduction

Recently a class of non-commutative non-cocommutative Hopf algebra has been found in the investigations of the integrable systems. These Hopf algebras are q -deformed function algebras of classical groups. This structure is called "quantum group" [Dri].

The structure of the quantum groups suggests the possibility of investigating a geometry where we can even consider discarding the commutativity of the algebra of coordinate functions. It is interesting to ask whether one can find applications of this new class of symmetry to some physical systems other than the integrable models.

Following this idea, the first step one has to make is to provide the appropriate tools for this investigation. To this end we take the usual application of the group theory as a guiding principle for the generalization of the q -deformed quantities. This is also useful since in the limit $q \rightarrow 1$ we wish to reproduce the results obtained in the ordinary classical group case.

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