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Stark Wannier Ladders

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Abstract. We study the Schrödinger equation for an electron in a one dimensional crystal submitted to a constant electric field. We prove the existence of ladders of resonances, the imaginary part of which is exponentially small with the field.

The Schrödinger equation for electrons in a crystal submitted to an external constant electrical field has attracted much attention [13] since it is a first step in understanding conductivity in solids. A recent review on the subject can be found in [11].

For several decades, the experimental evidence of resonance states (called also Bloch oscillators), was questioned. In fact, it was only recently, that their effect clearly appeared in the electro-optical properties of semiconductor superlattices (man-made crystals in which layers of two distinct semi-conductors alternate, the period in the perpendicular direction to the layers can be of the order of hundreds of normal lattice periods) [4, 12]. As it will be shown, resonant states live in regions whose length is proportional to the spectral band widths of the Bloch Hamiltonian and inversely proportional to the external field. So, occurrence of small energy bands near the Fermi energy, as in superlattices, favour their observation.

Mathematically, existence of resonances for the one dimensional Hamiltonian $-\frac{h^2}{2m}\frac{d^2}{dx^2} + V_P(x) + Fx$ has been rigorously proven for large external electric fields by Agler and Froese [1] in the case $V_P(x)$ is a Fourier series with a finite number of terms (F is the product of the particle charge by the electrical field). Nothing was said in this paper about the resonance widths which were expected to be exponentially small with respect to the electrical field (see the numerical treatment of the semi infinite Kronig-Penney model [2]).

In this paper, we give a new proof for the existence of the resonances, establish the link between their widths and the spectral properties of the Bloch Hamiltonian, and prove their exponential behavior. The localisation of the resonance states is understood in the scope of the tilted bands picture introduced by Zener. We shall