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Propagation Estimates for N-body Schroedinger Operators

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Abstract. We prove propagation estimates (of strong type) for long-range N-body Hamiltonians. Emphasis is on phase–space analysis in the free channel region.

1. Introduction

In this paper we prove various propagation estimates for a fairly large class of long-range N-body Schroedinger operators (denoted by H). The form of these estimates (in the configuration space representation) is

$$B(t)e^{-itH}f(H)\langle x\rangle^{-s'} = O(t^{-s}) \tag{1.1}$$

for $t \to +\infty$, and with $\langle x\rangle$ given by multiplication by $(1+|x|^2)^{1/2}, f \in C_0^\infty(\mathbf{R})$, $0 \leqq s < s'$, and finally with $\{B(t)\}_{t>0}$ a family of pseudodifferential operators (typically of non-negative order).

Among our estimates are the minimal and large velocity estimates of Sigal and Soffer [S–S] (obtained by putting $B(t) = \chi\left(\frac{x^2}{4t^2} - E < -\varepsilon\right)$ or $B(t) = \langle x\rangle^{s'-s}\chi\left(E' - \frac{x^2}{4t^2} < -\varepsilon\right)$, with $0 < E \ll E'$ and assuming f to be supported in a small neighbourhood of E, respectively), however obtained for arbitrary s and s' as above and for a larger class of potentials.

We shall also prove maximal velocity estimates for the free channel, which are obtained by localizing further in the configuration space namely to regions where the potential "goes to zero." Finally if χ_{fr} denotes such localization operator, we shall prove the estimate with $B(t) = P_-(X, D)\chi_{fr}$, where the symbol $p_-(x, \xi)$ vanishes in a certain conical neighbourhood of the forward direction: $x = c\xi, c > 0$. The latter result was established for $N = 2$ by Isozaki [I] and independently by Jensen [J] (in both cases under more restrictive conditions on the potential). For $N > 2$ the most resembling results in the literature seem to be due to Mourre