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$(T^*G)_t$: A Toy Model for Conformal Field Theory*

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Abstract. We study a chiral operator algebra of conformal field theory and quantum deformation of the finite-dimensional Lie group to obtain the definition of $(T^*G)_t$ and its representation.

The closeness of the Kač-Moody algebras, constituting the chiral operator algebra of a typical (and generic) conformal field theory model, namely the WZNW model, and quantum deformation of corresponding finite-dimensional Lie group G has become more and more evident in recent years [1–5]. This in particular prompts further investigation of the differential geometry of such deformations. The notion of tangent and cotangent bundles is basic in classical differential geometry. It is only natural that the quantum deformations of TG and T^*G are to be introduced alongside those for G itself. Physical ideas could be useful for this goal.

Indeed, the T^*G can be interpreted as a phase space for a kind of a top, generalizing the usual top associated with $G=SO(3)$. The classical mechanics is a natural language to describe differential geometry, whereas the usual quantization is nothing but the representation theory.

In this paper we put corresponding formulas in such a fashion that their deformation becomes almost evident, given the experience in this domain. As a result we get the definition of $(T^*G)_t$ and its representation (t is the deformation parameter).

To make the exposition most simple and formulas transparent we shall work on an example of $G=sl(2)$ and present results in such a way that the generalizations become evident. We shall stick to generic complex versions, real and especially compact forms requiring some additional consideration, not all of which are self-evident.

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