Commun. Math. Phys. 141, 293–327 (1991)

Communications in
Mathematical
Physics

Free Fermions and the Alexander-Conway Polynomial

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Received September 20, 1990; in revised form December 28, 1990

Abstract. We show how the Conway Alexander polynomial arises from the q deformation of (Z_2 graded) $sl(n,n)$ algebras. In the simplest $sl(1,1)$ case we then establish connection between classical knot theory and its modern versions based on quantum groups. We first show how the crystal and the fundamental group of the complement of a knot give rise naturally to the Burau representation of the braid group. The Burau matrix is then transformed into the $U_q sl(1,1)$ R matrix by going to the exterior power algebra. Using a det = str identity, this allows us to recover the state model of [K2, 89] as well. We also show how the $U_q sl(1,1)$ algebra describes free fermions "propagating" on the knot diagram. We rewrite the Conway Alexander polynomial as a Berezin integral, and thus as an apparently new determinant.

Introduction

This paper discusses the role of the Alexander-Conway polynomial [A23, C70] in relation to quantum groups and statistical mechanics. In particular, we show that a state model for this polynomial is identical in form to the free-fermion model in statistical mechanics. This means that the polynomial can be expressed as a discrete Berezin integral, and through this as a determinant that is distinct from the classical determinant definition of this polynomial.

A number of significant interconnections arise from our work. The Yang-Baxter solution in the $sl(1/1)$ case is seen to arise directly from the Burau representation of the Artin braid group (via its action on exterior powers). Thus we provide a direct line from the classical knot theory to the statistical mechanics. The

* Work supported in part by NSF grant no. DMS-8822602
** Work supported in part by the NSF: grant nos. PYI PHY 86-57788 and PHY 90-00386 and by CNRS, France
*** On leave from SPhT CEN Saclay, F-91191 Gif sur Yvette Cedex, France