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# Separation of Phases at Low Temperatures in a One-Dimensional Continuous Gas

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**Abstract.** We show the existence of a phase separation at low temperatures in a one-dimensional one-component classical gas in the canonical ensemble with interaction *hard core* $-1/r^{\alpha}$, $1<\alpha\leqq 2$. This implies that for sufficiently low temperatures there are values of the chemical potential at which the pressure is not differentiable as a function of the chemical potential.

## 0. Introduction

Most of the results on phase transitions in continuous models are for phase separation in mixtures and, to the author's knowledge, there are no results on the existence of a phase transition in a one-component classical continuous gas, see however Israel [1]. Extending ideas developed in Johansson [2] we will prove that a one-dimensional continuous gas in the canonical ensemble with attractive pair-interaction $1/r^{\alpha}$, $1<\alpha\leqq 2$, and a hard core has a phase transition at sufficiently low temperatures.

In the proof we rewrite the partition function for the continuous model as an integral of partition functions for discrete models. These discrete models are similar to a one-dimensional lattice gas in the canonical ensemble.

In the first section we define the model and state our results. The second section contains the representation of the continuous model as an integral of discrete models, the definition of blocks, partitions, and the rearrangement procedure and the main steps in the energy-entropy argument. In Sect. 3 and 4 we prove the basic entropy and energy estimates.

Many arguments in this paper are similar to the corresponding arguments in Johansson [2], which we will refer to as [I].

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