

Hochschild and Cyclic Homology of Quantum Groups

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Received October 23, 1990; in revised form February 20, 1991

Abstract. For an arbitrary complex linear semisimple Lie group G , we consider Hopf algebras of the deformations of the formal and algebraic functions on G . The Hochschild and cyclic homology of these Hopf algebras are computed when the value of the deformation parameter is generic.

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1. Introduction

According to Gelfand, the category of locally compact topological spaces is equivalent to the category of commutative C^* -algebras. Also, the category of affine

The main result of this paper was announced at the AMS meeting at University Park in April 1990

* Supported by Department of Energy under Grant DE-FG02-88ER25062

** Supported by Harvard Prize Fellowship. Address after July, 1990: Department of Mathematics, Pennsylvania State University, University Park, PA 16802, USA