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Hydrodynamic Limit for Attractive Particle Systems on $\mathbb{Z}^d$

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Abstract. We study the hydrodynamic behavior of asymmetric simple exclusions and zero range processes in several dimensions. Under Euler scaling, a nonlinear conservation law is derived for the time evolution of the macroscopic particle density.

1. Notation and Summary

In this article, we study the hydrodynamic behavior of certain stochastic particle systems, such as simple exclusions and zero range processes. These systems consist of an infinite number of identical particles that move on a multidimensional lattice according to a Markovian law. Under Euler scaling, the microscopic particle density converges to a deterministic limit that is characterized as the solution of a nonlinear conservation law.

Before stating our main results we describe the simple exclusion model and the zero-range process is more detail.

Let E denote the space of configurations $\eta = (\eta(u): u \in \mathbb{Z}^d)$, where $\eta(u)$ is a nonnegative integer representing the occupation number of particles at site u. Let $(p(z): z \in \mathbb{Z}^d)$ be a probability transition function $\left(\text{i.e. } \sum_z p(z) = 1 \text{ and } p(z) \geqq 0\right)$ and $g: \mathbb{N} \to [0, \infty)$ be a bounded nondecreasing function with $0 = g(0) < g(1)$. The zero-range processes are defined as Markov processes with state space E and generator

$$\mathscr{L} f(\eta) = \sum_{u,v} p(v-u) g(\eta(u))(f(\eta^{uv}) - f(\eta)),$$

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