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Stabilization of Needle-Crystals by the Gibbs–Thomson Effect

C.-A. Pillet*

Département de Physique Théorique, Université de Genève, CH-1211 Geneva 4, Switzerland

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Abstract. We develop a scheme based on pseudo-differential operators to analyze the propagation of excitations in inhomogeneous extended systems. This method is used in a very specific situation, however we think that it has some generality and should apply to various other problems of current interest. We study the well known two-dimensional *symmetric model* of solidification introduced by Langer and Turski. Assuming the existence of Ivantsov-like steady-state solutions, we calculate their excitation spectrum. We show that there are no unstable propagating modes if the Gibbs–Thomson effect is taken into account. This proves that the growth of needle-crystals is stable with respect to side-branching.

1. Introduction

During the last decade tremendous efforts have been made by both experimentalists and theoreticians to understand the dynamics of pattern forming systems in such various fields as hydrodynamics, reactions kinetics or aggregation processes. Despite the apparent variety of mechanisms involved in these phenomena some unifying features are now emerging from these works. There remain a lot of open questions and unsolved problems, in particular regarding the mathematical status of the theory, nevertheless a systematic approach to this kind of problem seems nowadays to be more than just a dream. We refer the reader to [L1–2] and [KKL] for comprehensive reviews of these subjects, and to [CE] for an nice introduction to the physics and mathematics of extended systems (i.e., systems that are not confined in some finite volume). In this paper we will be concerned with a specific problem that typically occurs in the study of extended systems: the determination of the continuous part of the excitation spectrum of a stationary (or periodic) state. This is an essential step in the study of such states since it allows to analyze its stability with respect to propagating excitations as opposed to localized modes which correspond to the discrete spectrum. The latter may be much more difficult

* Present address: Department of Mathematics, University of Toronto, Toronto Canada, M5S1A1