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Anomalies and Curvature of W Manifolds

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Abstract. We study the holomorphic structure of certain complex manifolds associated with W_∞ algebras, namely, the flag manifolds W_∞/T_∞ and $W_{1+\infty}/T_{1+\infty}$, and the spaces $W_\infty/SL(\infty,R)$ and $W_{1+\infty}/GL(\infty,R)$, where T_∞ and $T_{1+\infty}$ are the maximal tori in W_∞ and $W_{1+\infty}$. We compute their Ricci curvature and show how the results are related to the anomaly-freedom conditions for W_∞ and $W_{1+\infty}$. We discuss the relation of these manifolds with extensions of universal Teichmüller space.

1. Introduction

An important problem in string theory is the search for a better understanding of its geometrical underpinnings, in the spirit of the beautiful interpretation of general relativity in terms of Riemannian geometry. It has been argued that a natural arena for addressing such geometrical issues is provided by the study of the manifold $\mathcal{M} \equiv \text{diff}(S^1)/S^1$ of complex structures on loop space related by reparametrisations [1]. This remarkable manifold proves to possess a natural Kähler structure [1], and it has been found that many statements concerning the consistency of string theory can be reformulated in terms of geometric data for $\mathcal{M}$ or related structures [1–5]. For example, the condition of nilpotency for the BRST charge Q (required for quantisation in the BRST formalism) is replaced by the requirement that a certain vector bundle over $\mathcal{M}$ have vanishing Ricci curvature [1,2].

In this paper, we show that this geometrical formalism admits very natural extensions when one replaces the algebra diff(S^1) (essentially the centreless Virasoro algebra) by certain higher-spin extended algebras which have been

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