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The Bi-Hamiltonian Structure of Fully Supersymmetric Korteweg-de Vries Systems

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Abstract. The bi-Hamiltonian structure of integrable supersymmetric extensions of the Korteweg-de Vries (KdV) equation related to the $N=1$ and the $N=2$ superconformal algebras is found. It turns out that some of these extensions admit inverse Hamiltonian formulations in terms of presymplectic operators rather than in terms of Poisson tensors. For one extension related to the $N=2$ case additional symmetries are found with bosonic parts that cannot be reduced to symmetries of the classical KdV. They can be explained by a factorization of the corresponding Lax operator. All the bi-Hamiltonian formulations are derived in a systematic way from the Lax operators.

1. Introduction

Recently there has been much interest in (super-)conformal field theories [1, 2] in context of studying string theories and statistical models of critical phenomena. Along with these studies one observes a growing interest in the supersymmetrization of the Korteweg-de Vries (KdV) equation [3–5]. It turned out that the Virasoro algebra and some of its extensions can be related to the second Hamiltonian structure of the KdV and KdV-like equations. This Hamiltonian structure is given by a set of Poisson brackets for the fundamental fields representing the Virasoro algebra [6]. In fact, starting from supersymmetric generalizations of the Virasoro algebra and the corresponding Hamiltonian structure it was possible to construct integrable supersymmetric extensions of the classical KdV equation [7, 8].

Related to the $N=1$ superconformal algebra two integrable cases of fermionic extensions of the KdV equation were found [3, 9]. The first case [3] turns out to admit a bi-Hamiltonian formulation, but it fails to be invariant relative to space-supersymmetric transformations. The second case [10] admits such an