Commun. Math. Phys. 139, 183–200 (1991)

Communications in
Mathematical
Physics

Isosystolic Inequalities and the Topological Expansion for Random Surface and Matrix Models

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Received November 3, 1990; in revised form February 4, 1991

Abstract. Using the isosystolic inequalities on Riemann surfaces, we prove that for many random surface or matrix models the radius of convergence of the perturbative series at fixed genus is independent of the genus. This result applies for instance to the dynamically triangulated random surface model in any dimension or to many matrix models with regular propagators in the superrenormalizable domain, for instance $\lambda\phi^3$ in dimension $d<6$, $(\lambda\phi^4+\sqrt{\lambda}\phi^3)$ in dimension $d<4$, and various other $P(\phi)_2$ models (in particular all those containing an odd power of ϕ). We hope that this result is a first step towards a more rigorous understanding of the genus dependence of surface models or of quantum gravity coupled with matter fields.

I. Introduction

The topological expansion in field theory goes back to the proposal by 't Hooft to study gauge theory with gauge group $SU(N)$ by means of a $1/N$ expansion. This is because the order g in this $1/N$ expansion is in perturbation theory the sum of all Feynman graphs of genus g [1]. Topological expansions of this kind can be written in general for quantum field theories in which the field is an N by N matrix field.

Each order in the $1/N$ expansion is a full perturbative series in its own right, with Feynman graphs equipped with the regular propagator $(p^2+1)^{-1}$. The n^{th} order of this series is made of relatively few graphs: for instance in one of the simplest models, namely the matrix model with $\mathrm{Tr}\,\phi^4$ interaction (which for simplicity we call the ϕ^4 model) there are at most K^n graphs of order n and fixed genus (in sharp contrast with the ordinary ϕ^4 perturbative series, in which there are about $K^n n!$ graphs at order n) [2]. In the series of papers [3–5] the matrix models in 0 or 1 dimension of space time were beautifully analyzed. The asymptotic number of planar graphs with n vertices as n gets large was in particular computed, hence the optimal value of the constant K for planar graphs was found.