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Topological Representations of the Quantum Group $U_q(sl_2)$

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Abstract. We define a topological action of the quantum group $U_q(sl_2)$ on a space of homology cycles with twisted coefficients on the configuration space of the punctured disc. This action commutes with the monodromy action of the braid groupoid, which is given by the R-matrix of $U_q(sl_2)$.

0. Introduction

In the free field representation of conformal field theory based on $SU(2)$ one is led to consider integrals of the form [1, 2]

$$\begin{aligned} G_C(w_1, \dots, w_s) = &\int_C f(z_1, \dots, z_r, w_1, \dots, w_s) \\ &\times \prod_{i<j} (z_i - z_j)^{2\nu} \prod_{i,j} (z_i - w_j)^{(1-n_j)\nu} \prod_{i<j} (w_i - w_j)^{\frac{1}{2}(1-n_i)(1-n_j)\nu} \\ &\times dz_1 \wedge \dots \wedge dz_r . \end{aligned} \tag{0.1}$$

In this formula $n_1, \dots, n_s$ are positive integers, f is a single valued meromorphic function, symmetric under permutations of the z-variables, with poles on the hyperplanes $\{z_i = w_j\}$. The parameter ν is equal to $1/k+2$ for the WZW model on $SU(2)$ at level k and is equal to p'/p for minimal models with central charge $1-6(p-p')^2/pp'$.

For each integration cycle C in the r^{th} homology group with coefficients in the local system given by the monodromy of the differential form in (0.1), G_C is a many valued analytic function on the space $\mathscr{C}_{1,\dots,1}(\mathbf{C}) = \{(w_1, \dots, w_s) \in \mathbf{C}^s \mid w_i \neq w_j \ (i \neq j)\}$. To compute its transformation under analytic continuation along paths exchanging the punctures w_i, one needs to know the monodromy action of the braid

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