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# Local Conformal Field Algebras

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**Abstract.** The local conformal field algebras with the multiplication corresponding to the regularized pointwise product of fields in the operator algebras of the quantum conformal field theory are investigated.

## Introduction

The scenery of the theory of groups, their representations and homogeneous spaces in view of the intensive interaction with mathematical physics deeply depends on changes in it. Thus, forming the spectrum of problems of modern quantum field theory (string field theory, conformal field theory, etc.) provoked the elaboration of a "daughter" rapidly developping discipline including infinite dimensional geometry, harmonic analysis on the infinite dimensional manifolds and the theory of representations of the infinite dimensional algebras and groups.

At the same time methods of quantum field theory penetrated into abstract mathematics (topological quantum field theory [1–4]). There exist some arguments for the hypothesis that quantum conformal field theory [5–8] would be useful in singularity theory. One should suppose that the operator formalism of quantum field theory would play the role analogous to that of cohomology theory in topology.

All these circumstances explain the appearance of this paper devoted to the algebraic aspects of QCFT as the choice of the approach to this theory based on infinite dimensional geometry [9–20]. We construct the $L$-algebra $L(C\,\mathrm{vir})$, whose elements belong to the Fock space over the universal deformation of the complex disc, in which the model of the Verma modules over the Virasoro algebra is realised [20]. Then we investigate the local conformal field algebras (LCFAs), which may be characterised as algebras of geometric objects on the complex plane valued in the $L$-algebra $L(C\,\mathrm{vir})$. The multiplication in the LCFAs corresponds to the regularized pointwise product of fields in the operator algebras of QCFT.

For pedagogical purposes we shall consider the simpler case of the $sl(2, C)$-invariant field theory before the conformal one throughout the text.