Simplex Equations and Their Solutions

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Abstract. We investigate an n-simplex generalization of the classical and quantum Yang–Baxter equation. For the case of $sl(2)$ we find the most general solution of the classical n-simplex equation for all n. These classical solutions can be quantized (in the sense of quantum group theory) for $n = 2, 3$ and we exhibit a quantum solution to the tetrahedron equations ($n = 3$). The classical nondegenerate solutions cannot be quantized for $n = 4$.

1. Introduction

Conformal field theory has recently emerged as a powerful unifying principle for seemingly disparate mathematical theories such as representations of loop groups, the theory of Riemann surfaces and moduli spaces, quantum groups, and three dimensional topology. This new field is far from completion as a rigorous mathematical theory but the general structure and the key ideas are rather clear. From the physical point of view, conformal field theory can be considered as a simple nontrivial example of a quantum field theory. Although quantum field theory has been extensively developed by physicists during the past several decades, the mathematical structure of quantum field theory has been thoroughly understood only in the cases of free field theories and conformal field theories. The latter have already yielded surprising and far-reaching results. It may therefore be expected that other more sophisticated quantum field theories will be related to correspondingly deeper questions in mathematics. One of the motivations of the present paper was the search for mathematical structures in other quantum field theories which have clear analogies to those arising in conformal field theory.

A simple and basic ingredient in conformal field theory (following from the consistency of the operator product expansion) is the triangular, or Yang–Baxter

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