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Unitarization of a Singular Representation of $SO(p, q)$

B. Binegar and R. Zierau

Department of Mathematics, Oklahoma State University, Stillwater, Oklahoma 74078-0613, USA

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Abstract. A geometric construction of a certain singular unitary representation of $SO_e(p,q)$, with $p+q$ even is given. The representation is realized geometrically as the kernel of a $SO_e(p,q)$-invariant operator on a space of sections over a homogeneous space for $SO_e(p,q)$. The K-structure of these representations is elucidated and we demonstrate their unitarity by explicitly writing down an $\mathfrak{so}(p,q)$-invariant positive definite hermitian form. Finally, we demonstrate that the annihilator in $\mathcal{U}[\mathfrak{g}]$ of this representation is the Joseph ideal, which is the maximal primitive ideal associated with the minimal coadjoint orbit.

1. Introduction

The irreducible unitary representations of a semisimple Lie group G fall into two basic classes; the tempered representations which enter the Plancheral decomposition of $L^2(G)$, and the "singular representations" which form the complement of the tempered representations in the full unitary dual of G. There are fairly uniform geometric constructions of the tempered representations that associate these representations with certain orbits of semisimple elements is the dual of the Lie algebra of G.

There is no such uniform scheme for constructing the singular unitary representations. A good geometric construction seems to be the procedure of Rawnsley, Schmid and Wolf ([R–S–W]) which uses indefinite harmonic theory to unitarize Dolbeault cohomology. However, the procedure works only in a narrow setting; it associates most of the unitary highest weight modules to elliptic coadjoint orbits. Other singular representations have geometric realizations. For example, the metaplectic representation is constructed by a quantization procedure known as the Kostant–Sternberg–Blattner method of moving polarizations (see [B]). There are also constructions using Howe's dual pair picture (e.g., [M]), and there are constructions using twister techniques (e.g. [E–P–W] and [N1]). Even so, most of the success in constructing singular representations has been limited to singular highest weight representations.