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Ornstein-Zernike Decay in the Ground State of the Quantum Ising Model in a Strong Transverse Field

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Abstract. We consider the quantum mechanical Ising ferromagnet in a strong transverse magnetic field in any number of dimensions, d. We prove that in the ground state the power law correction to the exponential decay of the two point function is $d/2$. The proof begins by writing the ground state as a classical system in one more dimension. (Thus the classical Ornstein-Zernike power of $(d-1)/2$ becomes $d/2$). We then develop a convergent polymer expansion and use the techniques of Bricmont and Fröhlich [5].

When a lattice spin system is away from any critical points, the truncated correlation functions usually decay exponentially. This exponential decay is typically accompanied by a power law correction, i.e., the decay goes as $\exp(-|x|/\xi)/|x|^p$. An interesting question is to determine this power p. For classical systems the "generic" power is $(d-1)/2$. This is known as Ornstein-Zernike decay and has been proved in a variety of models by a variety of methods. (The literature in the classical case is vast. Some of the early references may be found in [5].) Now consider the ground state of a quantum mechanical spin system that is not critical. (For example, take the quantum mechanical Ising model in a strong transverse magnetic field.) This ground state is like a classical system in one more dimension, so the Ornstein-Zernike decay would be a power of $d/2$.

We prove that the power is indeed $d/2$ for one of the two point functions in the quantum mechanical Ising model in a strong magnetic field. The Hamiltonian of this model is

$$H=\sum_j (1-\sigma_j^z)-\varepsilon \sum_{\langle ij\rangle} \sigma_i^x \sigma_j^x \tag{2.1}$$

with ε small. When ε is sufficiently small this model has been proven to have a unique ground state [18]. We only consider the two point function $\langle \sigma_i^x \sigma_j^x \rangle$.

Our proof begins by using the Trotter product formula to write the ground state of the quantum system as a classical system in one more dimension. An early use of this now standard technique is Ginibre's proof of the existence of long range