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Characterization of States of Infinite Boson Systems

II. On the Existence of the Conditional Reduced Density Matrix

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Abstract. In the present paper we deal with the problem of existence and uniqueness of the conditional reduced density matrix (c.r.d.m.) corresponding to a locally normal state of a boson system. The c.r.d.m. was introduced in [3] (Part I of the present series of papers). In order to characterize the class of states possessing a c.r.d.m. we will introduce the family of conditional states of a locally normal state, and we will discuss the relation between the conditional states, the c.r.d.m. and the conditional distribution of the position distribution of the state.

1. Introduction

In [2, 3] we introduced the position distribution Q_ω and the conditional reduced density matrix k_ω (c.r.d.m.) of a locally normal state ω of a boson system. It was shown that Q_ω and k_ω determine the whole state. In the present paper we will characterize a class of states which possess a c.r.d.m. For that reason we first will introduce the notion of conditional states ω_Λ^φ of a state ω that describe the behaviour of the system inside a bounded region Λ having fixed a configuration φ outside this area. It is shown that the position distribution of the conditional state ω_Λ^φ is just the conditional distribution $Q_\omega(\cdot\,|_{\Lambda^c}\mathfrak{M})(\varphi)$ of the position distribution Q_ω.

Further, we will see that the c.r.d.m. exists if for each $\Lambda \in \mathfrak{B}$ the family (ω_Λ^φ) of conditional states exists and if Q_ω is Σ_v'-point process. Moreover, we will prove that the c.r.d.m. is a.e.-uniquely determined.

We use the notations and notions given in Part I [3]. We refer to this part by adding I, e.g. I.2.1 means Sect. 2.1 in [3].

As in the previous papers we consider exclusively locally normal states of bosons without spin. The phase space G is assumed to be Polish endowed with a locally finite diffuse measure v, and the local algebras consist of all bounded linear operators on the Fock space over the bounded regions of the Fock space G.