Commun. Math. Phys. 137, 427–459 (1991)

Communications in Mathematical Physics

The Asymptotics of the Arakelov-Green's Function and Faltings' Delta Invariant

R. Wentworth*

Department of Mathematics, Harvard University, Cambridge, MA 02138, USA

Received April 28, 1989; in revised form July 2, 1990

Abstract. We study the behavior of the Arakelov-Green's function and Faltings' delta invariant on degenerating Riemann surfaces.

1. Introduction

The analysis of the behavior of metrics, Green's functions, eigenvalues, and determinants of Laplace operators on spaces of degenerating Riemann surfaces has been carried out in many different contexts. Given a compact Riemann surface, one makes a particular choice from the conformal class of metrics compatible with the complex structure associated to the surface, and a fundamental problem is to relate the geometrical data arising from the metric to the complex analytic structure of the underlying surface. The metrics chosen, however, have nearly always been those with constant curvature. In this paper, we consider degeneration phenomena with respect to a different choice; namely, the Arakelov metric [2, 14, 21, 23].

One way to describe this metric is as follows: the Riemann surface M is embedded into its Jacobian variety $J(M)$ via the Abel map I. The canonical metric on M is the pullback by I of the flat metric on $J(M)$ induced by the polarization of $J(M)$ as an abelian variety, and the Arakelov metric can be defined by prescribing that its curvature be proportional to the Kähler form of the canonical metric. This only determines the metric up to a constant, but there is a second quantity associated to the canonical metric – the Green's function. We can fix the scaling of the Arakelov metric by requiring the logarithm of the distance in this metric to be exactly the singularity of the Green's function along the diagonal.

Spaces of degenerating surfaces correspond to paths in moduli space leading to the boundary points. These are obtained from compact surfaces by shrinking

* Supported in part by an Alfred P. Sloan Doctoral Dissertation Fellowship