Commun. Math. Phys. 137, 383–398 (1991)

Communications in Mathematical Physics

Algebraic Geometric Solutions of Einstein's Equations: Some Physical Properties

D. A. Korotkin

Department of Mathematics, Leningrad Institute of Aircraft Instrumentation, Gertzena, 67, SU-190000 Leningrad, USSR

Received June 14, 1990; in revised form October 29, 1990

Abstract. The physical properties of algebraic geometric solutions of stationary axisymmetric vacuum Einstein's equations are discussed. It appears that these solutions describe an interaction of a few localized rotating string-like objects on an arbitrary static background. If such an object collapses to a point then it produces a Kerr-NUT black hole.

Introduction

Recently the author and V.B. Matveev have shown [1–3] that the technique of multidimensional theta-functions first used by S.P. Novikov, B.A. Dubrovin, V.B. Matveev, A.R. Its, and I.M. Krichever [4–6] for finding periodic and almost periodic solutions of Korteveg de Vries (KdV), Non-linear Schroedinger (NSch) and other equations solvable by the inverse scattering method may be successfully applied to a stationary axisymmetric Einstein's equation in a vacuum and to the Einstein-Maxwell system. Further in [7, 8] the technique of [1–3] was extended to the four-dimensional self-duality equations for $SU(2)$ and $SU(1,1)$ groups.

The main properties of finite-gap solutions from [1–3] differ radically from the properties of well-known finite-gap periodic and almost periodic solutions of KdV, NSch and others. Namely the dynamics in finite-gap solutions of Einstein's equations is set by the deformation of the algebraic curve with variable branch points in contrast to the dynamics of the usual finite-gap solutions set by the linear flow on the Jacobi manifold of the fixed curve. As a consequence the solutions from [1–3] are not periodic but localized as degenerated-soliton solutions. It appeared possible to introduce in these solutions the functional parameters corresponding to an arbitrary static background.

This paper is devoted to the investigation of most elementary physical properties of non-degenerated finite-gap solutions of vacuum stationary axisymmetric Einstein's equation found in [1–3]. The general genus g solution of stationary axisymmetric Einstein's equation describes an interaction of g localised