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Integration of the Nonlinear Schroedinger Equation with a Self-Consistent Source

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Abstract. It is shown that the nonlinear Schroedinger equation with a self-consistent source admits investigation by the inverse scattering method for the Dirac operator. The conditions are found under which the solutions of the nonlinear Schroedinger equation with a self-consistent source describe the creation and annihilation of solitons.

1. Introduction

At present, in the investigation of nonlinear evolution equations by the inverse scattering method there comes into view a new perspective trend: the case in point is the application of this method to integration of nonlinear evolution equations with a source. Being different in details, the use of the inverse scattering method for integrating different nonlinear evolution equations with a source has much in common both in the scheme of integration and in the dynamics of the obtained solutions. In this paper, these statements will be exemplified by the nonlinear Schroedinger equation with a self-consistent source. More precisely, we consider the integration of the following system of equations:

$$i\frac{\partial u}{\partial t} + 2|u|^2 u + \frac{\partial^2 u}{\partial x^2} = 2i\sum_{n=1}^{N}(\varphi_n p_n - \bar{\psi}_n \bar{q}_n), \tag{1}$$

$$\frac{\partial \varphi_n}{\partial x} + u\psi_n - i\zeta_n\varphi_n = \frac{\partial \psi_n}{\partial x} - \bar{u}\varphi_n + i\zeta_n\psi_n = 0, \qquad n = 1, \ldots, N, \tag{2}$$

$$\frac{\partial p_n}{\partial x} + uq_n - i\zeta_n p_n = \frac{\partial q_n}{\partial x} - \bar{u}p_n + i\zeta_n q_n = 0, \qquad n = 1, \ldots, N, \tag{3}$$

where the bar means complex conjugation. We shall assume that the function $u = u(x, t)$ at any $t \geq 0$ satisfies the requirement

$$\sum_{r=0}^{2}\int_{-\infty}^{\infty}\left|\frac{\partial^r u(x, t)}{\partial x^r}\right| dx < \infty. \tag{4}$$