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Semi-Global Existence and Convergence of Solutions of the Robinson-Trautman (2-Dimensional Calabi) Equation

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Abstract. It is shown that for smooth initial data solutions of the Robinson-Trautman equation (also known as the two-dimensional Calabi equation) exist for all positive "times," and asymptotically converge to a constant curvature metric.

1. Introduction

One of the most interesting problems in classical general relativity is the understanding of the formation – or lack thereof – of singularities in space-time in the course of evolution of the metric via Einstein equations. Since the tools to analyze this problem in its whole generality do not seem to be available yet, it is of interest to try to understand such issues under various restrictive hypotheses, e.g. smallness of initial data (cf. e.g. [6, 11]), or under some symmetry hypotheses [5, 20, 16, 8, 7], or both, or under some other restrictions (cf. e.g. [9]). In this paper we prove semi-global existence and convergence for a class of vacuum solutions of Einstein equations known as the Robinson-Trautman metrics [25], thus no curvature singularities (other than the singularity $r=0$ which is already present in the initial data set) develop for finite values of the retarded time[1] u in this class of metrics. The Robinson-Trautman metrics have played an important role in the early understanding of gravitational radiation, providing the first known class of solutions of Einstein vacuum equations which could be interpreted as representing a gravitationally radiating isolated system. These metrics have the amusing

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[1] More precisely, no curvature singularities develop for finite positive values of u if the total mass is positive, for negative values of u if the total mass is negative. It should be noted that the space-times the existence of which is established here will still be singular in the sense of being geodesically incomplete, cf. [28] and Proposition 2.1.