Commun. Math. Phys. 137, 263–287 (1991)

Communications in
Mathematical
Physics

A Renormalization Group Analysis of the Kosterlitz–Thouless Phase

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Received July 5, 1990

Abstract. We consider a classical Coulomb gas with a short distance cutoff in two dimensions; equivalently a Sine–Gordon field theory. For low temperature β^{-1} and low activity z the gas is in a multipole phase, the Kosterlitz–Thouless phase. For $\beta > 8\pi$ and z sufficiently small we give a complete renormalization group analysis for this phase and show that the flow of the effective measures is toward a free field (infrared asymptotic freedom). This should lead to control over the long distance behavior of the theory.

1. Introduction

A classical Coulomb gas in two dimensions with inverse temperature β and activity z is defined by the grand canonical partition function

$$\sum_{n=0}^{\infty} z^n/n! \left[\sum_{q_1,\ldots,q_n} \int dx_1 \cdots dx_n \exp\left(-\beta/2 \sum_{i,j} q_i q_j v(x_i - x_j) \right) \right], \tag{1.1}$$

where the sum is over charges $q_i = \pm 1$. The potential $v(x-y)$ is the inverse Laplacian $(\Delta^{-1})(x,y)$ with a short distance cutoff (essential for β large). An equivalent expression is as a Sine–Gordon field theory, namely

$$\int \exp(-2z \int \cos \varphi(x) dx) d\mu_{\beta v}(\varphi), \tag{1.2}$$

where $\mu_{\beta v}$ is a massless Gaussian measure with covariance βv.

For a dilute gas (z small) there is a phase transition as the temperature is lowered. At high temperatures (β small) there is a plasma phase with Debye screening and exponential decay of correlations. This was rigorously established by Brydges and Federbush [BF] and Yang [Y]. At low temperatures (β large) there is a multipole phase characterized by a power law decay of correlations. This

* Research supported by the Natural Sciences and Engineering Research Council