Commun. Math. Phys. 137, 231–247 (1991)

Communications in Mathematical Physics

An Algorithm for Detecting Abelian Monopoles in SU_2-Valued Lattice Gauge-Higgs Systems

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Received March 20, 1990; in revised form September 11, 1990

Abstract. We present an algorithm which calculates the monopole number of an SU_2-valued lattice gauge field, together with a lattice Higgs field, on a simplicial lattice of dimension $\geqq 3$. The calculation is gauge invariant. The expected value of the monopole density (for a fixed Higgs field) does not depend on the Higgs field.

Introduction

This paper addresses the problem of locating the abelian monopoles of an SU_2-valued lattice gauge-Higgs field system on a complex of dimension $\geqq 3$. In the smooth case, these phenomena have been extensively studied from the analytic and algebraic-geometric side (for example, [2–6, 9, 16]) but we believe a more local and topological analysis, besides being of intrinsic interest, will be useful in the study of monopoles as vacuum fluctuations, especially by lattice-theoretic methods as in [7]. A summary of this material appeared in [12].

We present an algorithm which, given a generic SU_2-valued lattice gauge field $\mathbf{u}$, and a lattice Higgs field $\mathbf{e}$, defined on a locally ordered simplicial lattice Λ, associates to each oriented 3-simplex $\Delta \in \Lambda$ an integer $\mu_{\mathbf{u},\mathbf{e}}(\Delta)$, the monopole number of the pair $\mathbf{u}, \mathbf{e}$ in Δ.

The lattice gauge field $\mathbf{u}$, as usual, assigns to every oriented 1-simplex $\langle ij \rangle \in \Lambda$ an SU_2-element u_{ij}, with $u_{ji} = u_{ij}^{-1}$, and the lattice Higgs field $\mathbf{e}$ assigns to each vertex $\langle i \rangle \in \Lambda$ a unit vector $e_i \in \mathbf{R}^3$. If we change gauge via a family $\mathbf{g} = \{g_i : \langle i \rangle \in \Lambda\}$ of elements of SU_2, then in the new gauge $\mathbf{u}$ becomes the lattice gauge field $\mathbf{gug}^{-1}$ which assigns $g_i u_{ij} g_j^{-1}$ to $\langle ij \rangle$, and $\mathbf{e}$ transforms under the adjoint action to $\mathbf{geg}^{-1}$, which assigns $g_i * e_i = g_i e_i g_i^{-1}$ to $\langle i \rangle$, identifying SU_2 with the unit quaternions and $\mathbf{R}^3 = \{a\mathbf{i} + b\mathbf{j} + c\mathbf{k}\}$ with the pure imaginary quaternions as usual.

* Partially supported by NSF grants DMS 8607168 and DMS 8907753
** Partially supported by PSC-CUNY and by NSF grant DMS 8805485