Commun. Math. Phys. 136, 607–623 (1991)

Communications in
Mathematical
Physics

Cyclic L-Operator Related with a 3-State R-Matrix

V. V. Bazhanov and R. M. Kashaev

Institute for High Energy Physics, Protvino, Moscow Region, USSR

Received May 29, 1990; in revised form July 15, 1990

Abstract. We consider the problem of constructing a cyclic L-operator associated with a 3-state R-matrix related to the $U_q(sl(3))$ algebra at $q^N = 1$. This problem is reduced to the construction of a cyclic (i.e. with no highest weight vector) representation of some twelve generating element algebra, which generalizes the $U_q(sl(3))$ algebra. We found such representation acting in $C^N \otimes C^N \otimes C^N$. The necessary conditions of the existence of the intertwining operator for two representations are also discussed.

0. Introduction

Recently, it was observed [1] that the chiral Potts model [2–4] can be considered as a part of some new algebraic structure related to the six-vertex R-matrix. In particular, the high genus algebraic relations between the Boltzmann weights of the chiral Potts model arise as a condition of the existence of an intertwining operator for two different representations of some quadratic Hopf algebra [5–7], which generalizes the $U_q(sl(2))$ algebra. This structure leads to various functional relations [1, 8], which completely determine the spectrum of the chiral Potts model transfer matrix. In fact, the largest eigenvalue was very recently calculated [9] using these functional relations.

It is natural to make an attempt to find new solvable lattice models whose Boltzmann weights obey high genus algebraic relations generalizing the results of [1] for the case of other R-matrices.

As a simplest possibility, one can replace the six-vertex R-matrix by the eight-vertex one. In this way one can discover [10] two cases of the integrable deformation of the chiral Potts model. The first case is, in fact, the deformation of Fateev-Zamolodchikov model [11] into the "broken Z_N-model" of [12]. The second case is an integrable deformation of the super-integrable chiral Potts model [13]. Incidentally, the former case was recently studied in [14].

In the present paper we consider the case of the three-state R-matrix of [15, 16, 20], which is related to the $U_q(sl(3))$ algebra with $q^N = 1$. As in the case