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Combinatorics of Representations of $U_q(\widehat{\mathfrak{sl}}(n))$ at $q=0$

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Abstract. The $q=0$ combinatorics for $U_q(\widehat{\mathfrak{sl}}(n))$ is studied in connection with solvable lattice models. Crystal bases of highest weight representations of $U_q(\widehat{\mathfrak{sl}}(n))$ are labelled by paths which were introduced as labels of corner transfer matrix eigenvectors at $q=0$. It is shown that the crystal graphs for finite tensor products of l-th symmetric tensor representations of $U_q(\mathfrak{sl}(n))$ approximate the crystal graphs of level l representations of $U_q(\widehat{\mathfrak{sl}}(n))$. The identification is made between restricted paths for the RSOS models and highest weight vectors in the crystal graphs of tensor modules for $U_q(\widehat{\mathfrak{sl}}(n))$.

1. Introduction

1.1 R Matrices and Paths. The eminent role of the quantized enveloping algebras in solvable lattice models is widely known. The R matrices, which are the intertwiners of tensor product representations, give the Boltzmann weights of lattice models with commuting transfer matrices [1].

Consider $U_q(\widehat{\mathfrak{sl}}(n))$. Let (V,π) be the l-th symmetric tensor representation of $U_q(\mathfrak{sl}(n))$. We can extend this representations to a family of representations (V,π_x) of $U_q(\widehat{\mathfrak{sl}}(n))$ with an auxiliary parameter x. The R matrix $R(x,y)$ is an element of $\mathrm{End}(V\otimes V)$ which intertwines two representations $(V\otimes V,\pi_x\otimes\pi_y)$ and $(V\otimes V,\pi_y\otimes\pi_x)$. Set

$$\mathscr{A}_l^+=\left\{v=\sum_{i=0}^{n-1}v_i\epsilon_i\,\middle|\,v_i\in\mathbf{Z}_{\geqq 0},\ \sum_{i=0}^{n-1}v_i=l\right\},$$

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