

Simple WZW Currents

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Abstract. A complete classification of simple currents of WZW theories is obtained. The proof is based on an analysis of the quantum dimensions of the primary fields. Simple currents are precisely the primaries with unit quantum dimension; for WZW theories explicit formulae for the quantum dimensions can be derived so that an identification of the fields with unit quantum dimension is possible.

1. Simple Currents

Simple currents are by definition primary fields Φ which upon taking fusion products simply permute the set of primary fields $\{\phi\}$ of a conformal field theory. They play a prominent role in the construction of modular invariant partition functions [1]. An important step in the classification of conformal field theories is therefore the enumeration of simple currents. For WZW theories [2, 3], a large class of simple currents is already known, namely the so-called cominimal fields [4] as well as an exceptional isolated case [5]. In the present paper, we prove that these known examples already exhaust the set of simple currents of WZW theories.

The distinctive property of simple currents which allows for their classification is the special value of their *quantum dimension*. Recall [6] that to any primary field ϕ of a two-dimensional conformal field theory, one can associate a quantum dimension $\mathcal{D}(\phi)$ which, loosely speaking, describes the relative size of the highest weight module of the chiral algebra carried by ϕ as compared to the highest weight module carried by the identity primary field $\mathbf{1}$. By a straightforward manipulation of characters, one can express $\mathcal{D}(\phi)$ through the matrix S that implements the modular transformation $\tau \rightarrow -\frac{1}{\tau}$ on the set of (characters of) primary fields as

$$\mathcal{D}(\phi) = \frac{S_{\phi\mathbf{1}}}{S_{\mathbf{1}\mathbf{1}}}. \quad (1.1)$$

From elementary properties of the modular matrix S , it follows that quantum dimensions are real numbers, that conjugate primary fields possess the same