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Local Rings of Singularities and $N = 2$ Supersymmetric Quantum Mechanics*

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Abstract. We investigate the Kähler structure arising in n-component, $N=2$ supersymmetric quantum mechanics. We define L^2-cohomology groups of a modified $\bar{\partial}$-operator and relate them to the corresponding spaces of harmonic forms. We prove that the cohomology is concentrated in the middle dimension, and is isomorphic to the direct sum of the local rings of the singularities of the superpotential. In the physics language, this means that the number of ground states is equal to the absolute value of the index of the supercharge, and each ground state contains exactly n fermions.

I. Introduction

$N=2$ supersymmetric Wess–Zumino models in one and two dimensions have been extensively studied over the past few years. These quantum field theory models are particularly rich in structure, and serve as a nontrivial example in studying the phenomenon of supersymmetry breaking [CG 1, 2, GIM], constructive field theory [JLW, JL 1], as well as string theory [GSW]. Wess–Zumino models are far from being exactly solvable. Yet, $N=2$ supersymmetry allows for closed form computations of various numerical characteristics of the models. The simplest of these characteristics is the index of the supersymmetry generator, the supercharge. The supercharge plays a similar role and has a similar structure as the Dirac operator in differential geometry. Its index is a topological invariant which captures certain qualitative features of the model, and is independent of its details (see e.g., Sect. II of [JL 2] for a precise formulation of this statement). It is known [JLL] that in one-component $N=2$ Wess–Zumino quantum mechanics with a polynomial superpotential, the number of ground states is equal to the index of the supercharge. It was also proven that this number is equal to the algebraic degree of the superpotential minus one.

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