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Hamiltonian Constructions of Kähler–Einstein Metrics and Kähler Metrics of Constant Scalar Curvature

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Abstract. Assuming the existence of a real torus acting through holomorphic isometries on a Kähler manifold, we construct an ansatz for Kähler–Einstein metrics and an ansatz for Kähler metrics with constant scalar curvature. Using this Hamiltonian approach we solve the differential equations in special cases and find, in particular, a family of constant scalar curvature Kähler metrics describing a non-linear superposition of the Bergman metric, the Calabi metric and a higher dimensional generalization of the LeBrun Kähler metric. The superposition contains Kähler–Einstein metrics and all the geometries are complete on the open disk bundle of some line bundle over the complex projective space $\mathbf{P}^n$. We also build such Kähler geometries on Kähler quotients of higher cohomogeneity.

Introduction

In this paper, we construct ansätze for Kähler geometries with commuting holomorphic isometries. We express the Kähler–Einstein condition and the condition for constant scalar curvature Kähler metrics as a system of differential equations with respect to the Kähler quotient coordinates and the Hamiltonian functions. This symplectic approach to special Kähler geometry with symmetry has recently been exploited most successfully by LeBrun [7-9]. Also, our concern with Kähler metrics of constant scalar curvature was inspired by the work of Calabi [2, 4].

In the case of Kähler metrics with circle action, we solve the equations under some reasonable geometrical assumptions and find, in particular, a family of Kähler geometries with constant negative scalar curvature on the open disk bundle of any complex line bundle $\mathcal{O}(-p)$ over the complex projective space $\mathbf{P}^n$ when p is sufficiently large. If one considers the open disk bundle to be differentiably

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